Tercer ejercicio de Quetzal U. O. Vargas H.

Ejercicio $\begin{bmatrix} 2 \\ 2.2 \\ 3 \end{bmatrix} \subset [2]$

Let $T: R^2 \longrightarrow R^3$ be defined by:

$T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$

Let β be the standard ordered basis for R^2 and:

$\gamma = \{(1,1,0), (0,1,1), (2,2,3)\}$

$\alpha = \{(1,2), (2,3)\}$

Compute a) $[T]_\beta^\gamma$ and b) $[T]_\alpha^\gamma$

a) Sea $T(a_1, a_2, a_3) = (a_1 - a_2, a_1, 2a_1 + a_2)$

$\beta = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

$T(1,0) = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ $T(0,1) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\gamma = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \right\}$

$$a_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + a_3 \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$b_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + b_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + b_3 \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Así tenemos la matriz aumentada:

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 2 & 1 & -1 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 2 & 1 \end{array}\right].$$

$(R_1)(-1)+R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 2 & 1 \end{array}\right]$$

$(R_2)(-1)+R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 3 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc:c:c} 1 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 3 & 2 & 0 \end{array}\right]$$

$(R_3)(\frac{1}{3}) \to R_3,\ (R_3)(-\frac{2}{3}) + R_1 \to R_1$

$$\begin{array}{c} \left[\begin{array}{ccc:c:c} 1 & 0 & 0 & -\frac{1}{3} & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{2}{3} & 0 \end{array}\right] \\ \begin{array}{ccccc} & & & \quad\quad\quad a_j & \quad b_j \end{array} \end{array}$$

$$\Rightarrow [T]_\beta^\gamma = \begin{bmatrix} -\frac{1}{3} & -1 \\ 0 & 1 \\ \frac{2}{3} & 0 \end{bmatrix}$$

b) Sea $T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$

$$\alpha = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$$

$$T \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \underline{\begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}} \qquad T \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \underline{\begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}}$$

$$\gamma = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \right\}$$

$$a_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + a_3 \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \underline{\begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}}$$

$$b_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + b_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + b_3 \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \underline{\begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}}$$

Así tenemos la matriz aumentada.

$$\left[\begin{array}{ccc:c:c} 1 & 0 & 2 & -1 & -1 \\ 1 & 1 & 2 & 1 & 2 \\ 0 & 1 & 3 & 4 & 7 \end{array}\right]$$

$(R_1)(-1)+R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc:c:c} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 4 & 7 \end{array}\right]$$

$(R_2)(-1)+R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc:c:c} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 3 & 2 & 4 \end{array}\right]$$

$(R_3)(\frac{1}{3}) \longrightarrow R_3$, $(R_3)(-\frac{2}{3})+R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc:c:c} 1 & 0 & 0 & -\frac{7}{3} & -1 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & \frac{2}{3} & \frac{4}{3} \end{array}\right]$$

$$\Rightarrow [T]_\alpha^\gamma = \begin{bmatrix} -\frac{7}{3} & -1 \\ 2 & 3 \\ -\frac{2}{3} & \frac{4}{3} \end{bmatrix}$$