$$J = \begin{pmatrix} 1 \\ 1_0 \, ? \\ 2 \end{pmatrix}$$

Cabiare el signo por letras

a) $(a, b, c, d) \to (a \cdot b,\ b - a,\ c,\ d)$

Supongamos $u = \begin{pmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{pmatrix}$ y $v = \begin{pmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{pmatrix}$

$$T(u+v) = \begin{pmatrix} a_1 + a_2 \\ b_1 + b_2 \\ c_1 + c_2 \\ d_2 + d_2 \end{pmatrix} = \begin{bmatrix} (a_1+a_2)\cdot(b_1+b_2) \\ (b_1+b_2) - (a_1+a_2) \\ c_1 + c_2 \\ d_1 + d_2 \end{bmatrix}$$

$$a_1 b_1 + a_1 b_2 + a_2 b_1 + a_2 b_2$$

Lo que vemos es que en la primer linea no podemos separar $L(u+v) = L(u) + L(v)$

$\therefore$ no es frastormacion lineal

b) $(a, b, c, d) \to (\lambda b,\ b - a,\ c,\ d)$

Supongamos $u = (a_1, b_1, c_1, d_1)$ y $v = (a_2, b_2, c_2, d_2)$

$$T(u+v) = \begin{pmatrix} a_1 + a_2 \\ b_1 + b_2 \\ c_1 + c_2 \\ d_1 + d_2 \end{pmatrix} = \begin{bmatrix} \lambda(b_1+b_2) \\ (b_1+b_2) - (a_1+a_2) \\ c_1 + c_2 \\ d_1 + d_2 \end{bmatrix} = \begin{bmatrix} \lambda b_1 + \lambda b_2 \\ b_1 - a_1 + b_2 - a_2 \\ c_1 + c_2 \\ d_1 + d_2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \lambda b_1 \\ b_1 - a_1 \\ c_1 \\ d_1 \end{bmatrix}}_{T(u)} + \underbrace{\begin{bmatrix} \lambda b_2 \\ b_2 - a_2 \\ c_2 \\ d_2 \end{bmatrix}}_{T(v)} = T(u) + T(v)$$

Supongamos ahora que α es escalar

$$T(cu) = T\begin{pmatrix} \alpha a \\ \alpha b \\ \alpha c \\ \alpha d \end{pmatrix} = \begin{bmatrix} \alpha \lambda b \\ \alpha b - \alpha a \\ \alpha c \\ \alpha d \end{bmatrix} = \begin{bmatrix} \alpha(\lambda b) \\ \alpha(b-a) \\ \alpha(c) \\ \alpha(d) \end{bmatrix} = \underbrace{\alpha \begin{bmatrix} \lambda b \\ b-a \\ c \\ d \end{bmatrix}}_{c\,Tu}$$

$\therefore$ Si es trasformacion lineal

d $(a, b, c, d) \to (0, c, b, a+b+c+d)$

Supongamos $u = (a_1, b_1, c_1, d_1)$ $V = (a_2, b_2, c_2, d_2)$

$$T(u+V) = \begin{pmatrix} a_1 + a_2 \\ b_1 + b_2 \\ c_1 + c_2 \\ d_1 + d_2 \end{pmatrix} = \begin{pmatrix} 0 \\ c_1 + c_2 \\ b_1 + b_2 \\ a_1 + a_2 + b_1 + b_2 + c_1 + c_2 + d_1 + d_2 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 0 \\ c_1 \\ b_1 \\ a_1 + b_1 + c_1 + d_1 \end{pmatrix}}_{T(u)} + \underbrace{\begin{pmatrix} 0 \\ c_2 \\ b_2 \\ a_2 + b_2 + c_2 + d_2 \end{pmatrix}}_{T(V)} = T(u) + T(V)$$

Supongamos α es escalar

$$T(\alpha u) = T\begin{pmatrix} \alpha(0) \\ \alpha c \\ \alpha b \\ \alpha(a+b+c+d) \end{pmatrix} = \alpha \underbrace{\begin{pmatrix} 0 \\ c \\ b \\ a+b+c+d \end{pmatrix}}_{T(u)} = \alpha T(u$$

$\therefore$ Si es trasformacion lineal