2. Let β and γ be the standard ordered bases for R^n and R^m, respectively. For each linear transformation $T: R^n \to R^m$, compute $[T]_\beta^\gamma$.

(a) $T: R^2 \to R^3$ defined by $T(a_1, a_2) = (2a_1 - a_2, 3a_1 + 4a_2, a_1)$

Tenemos la transformación definida como:

$$T(a_1, a_2) = (2a_1 - a_2,\ 3a_1 + 4a_2,\ a_1)$$

Entonces:

$$T(1,0) = (2,3,1) = 2e_1 + 3e_2 + 1e_3$$

y

$$T(0,1) = (-1,4,0) = -1e_1 + 4e_2 + 0e_3$$

Por lo tanto tenemos

$$[T]_\beta^\gamma = \begin{pmatrix} 2 & -1 \\ 3 & 4 \\ 1 & 0 \end{pmatrix} //$$

(b) $T: R^3 \to R^2$ defined by $T(a_1, a_2, a_3) = (2a_1 + 3a_2 - a_3, a_1 + a_3)$.

$$T(a_1, a_2, a_3) = (2a_1 + 3a_2 - a_3, a_1 + a_3)$$

Entonces:

$$T(1,0,0) = (2,1) = 2e_1 + 1e_2$$
$$T(0,1,0) = (3,0) = 3e_1 + 0e_2$$
$$T(0,0,1) = (-1,1) = -1e_1 + 1e_2$$

Por lo tanto tenemos:

$$[T]_\beta^\gamma = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}$$

(c) $T: R^3 \to R$ defined by $T(a_1, a_2, a_3) = 2a_1 + a_2 - 3a_3$

$$T(a_1, a_2, a_3) = (2a_1 + a_2 - 3a_3)$$

Entonces:

$$T(1,0,0) = (2) = 2e_1$$
$$T(0,1,0) = (1) = 1e_1$$
$$T(0,0,1) = (-3) = -3e_1$$

Por lo tanto tenemos:

$$[T]_\beta^\gamma = (2\ 1\ 3)$$

(d) $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(a_1, a_2, a_3) = (2a_2 + a_3, -a_1 + 4a_2 + 5a_3, a_1 + a_3)$

$$T(a_1, a_2, a_3) = (2a_2 + a_3, -a_1 + 4a_2 + 5a_3, a_1 + a_3)$$

Entonces:

$$T(1,0,0) = (0,-1,1) = 0e_1 - 1e_2 + 1e_3$$
$$T(0,1,0) = (2,4,0) = 2e_1 + 4e_2 + 0e_3$$
$$T(0,0,1) = (1,5,1) = 1e_1 + 5e_2 + 1e_3$$

Por lo tanto tenemos:

$$[T]_\beta^\gamma = \begin{pmatrix} 0 & 2 & 1 \\ -1 & 4 & 5 \\ 1 & 5 & 1 \end{pmatrix} //$$

(e) $T: \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(a_1, a_2, \ldots, a_n) = (a_1, a_1, \ldots, a_1)$

$$T(a_1, a_2, \ldots, a_n) = (a_1, a_1, \ldots, a_1)$$

Entonces:

$$T(e_1) = T\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \quad T(e_2) = T\begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \ldots$$

$$\ldots, T(e_n) = T\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \implies \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ \vdots & & \ddots & & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix} //$$

(f) $T: R^n \to R^n$ defined by $T(a_1, a_2, \ldots, a_n) = (a_n, a_{n-1}, \ldots, a_1)$

$$T(a_1, a_2, \ldots, a_n) = (a_n, a_{n-1}, \ldots, a_1)$$

Entonces:

$$T(e_1) = T\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \quad T(e_2) = T\begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \end{pmatrix}, \ldots$$

$$\ldots, T(e_n) = T\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{pmatrix} //$$

(g) $T: R^n \to R$ defined by $T(a_1, a_2, \ldots, a_n) = a_1 + a_n$

$$T(a_1, a_2, \ldots, a_n) = a_1 + a_n$$

Entonces:

$$T(e_1) = T\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}, \quad T(e_2) = T\begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}, \ldots$$

$$\ldots, T(e_n) = T\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & \ldots & 0 & 1 \end{pmatrix} //$$