(2.6, 4 c(2))

4) $T : M_{2\times3}(F) \to M_{2\times2}(F)$

$$T\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{pmatrix}$$

sea $U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \end{pmatrix}$ $V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \end{pmatrix}$ $\in M_{2\times3}(F)$

$$T(V+U) = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \end{pmatrix} \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \end{pmatrix}$$

$$= \begin{pmatrix} V_{11}+U_{11} & V_{12}+U_{12} & V_{13}+U_{13} \\ V_{21}+U_{21} & V_{22}+U_{22} & V_{23}+U_{23} \end{pmatrix}$$

$$= \begin{pmatrix} 2V_{11}+2U_{11} - V_{12}+U_{12} & V_{13}+U_{13} + 2V_{12}+2U_{12} \\ 0 & 0 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} (2V_{11} - V_{12}) & (2V_{11}, V_{13}) \\ 0 & 0 \end{pmatrix}}_{= T(V)} \underbrace{\begin{pmatrix} (2U_{11} - U_{12}) & (U_{13} + 2U_{12}) \\ 0 & 0 \end{pmatrix}}_{T(U)}$$

sea $\lambda \in (F)$ y $U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \end{pmatrix} \in M_{2\times3}(F)$

$$T(\lambda U) = \lambda T(U)$$

$$= T\left(\lambda \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \end{pmatrix}\right)$$

$$= T\begin{pmatrix} \lambda U_{11} & \lambda U_{12} & \lambda U_{13} \\ \lambda U_{21} & \lambda U_{22} & \lambda U_{23} \end{pmatrix}$$

$$\begin{pmatrix} 2\lambda U_{11} - \lambda U_{12} & \lambda U_{13} + 2\lambda U_{12} \\ 0 & 0 \end{pmatrix}$$

$$\lambda \begin{pmatrix} 2U_{11} - V_{12} & U_{13} + 2U_{12} \\ 0 & 0 \end{pmatrix}$$

$\therefore$ T es una transformacion lineal

sea $A = \begin{pmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{pmatrix}$

$nul(A) = 0$ solo hay un pivote en la matriz

$R(A) = 1$

$\therefore Dim(A) = 1$