$$F = \left\{ \begin{pmatrix} 10 \\ 10.2 \\ 10 \end{pmatrix} \right\} \subset [6] \qquad L: \mathbb{R}^4 \to \mathbb{R}^4$$

(b) Escalonamos la matriz

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 1 & -1 & 2 \\ 1 & 0 & 0 & -1 \\ 4 & 1 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -3 & -3 & -4 \\ 0 & -2 & -1 & -4 \\ 0 & -7 & -5 & -12 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & 4/3 \\ 0 & 0 & 1 & -4/3 \\ 0 & 0 & 2 & -8/3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & 4/3 \\ 0 & 0 & 1 & -4/3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Dado que A tiene 3 pivotes, entonces

$\text{Ran}(A) = 3$

$\text{Im}(A) = \text{gen}\{(1,2,1,3), (0,1,1,4/3), (0,0,1,-4/3)\}$

a) Ahora reducimos la matriz aumentada

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 4/3 & 0 \\ 0 & 0 & 1 & -4/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & 13/3 & 0 \\ 0 & 1 & 0 & 8/3 & 0 \\ 0 & 0 & -1 & 4/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 8/3 & 0 \\ 0 & 0 & 1 & -4/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

De la última simplificación, obtenemos

$$\begin{aligned} x &= w \\ y &= -8w/3 \\ z &= 4w/3 \\ w &= w \end{aligned} \Rightarrow \begin{pmatrix} w \\ -\frac{8w}{3} \\ \frac{4w}{3} \\ w \end{pmatrix} = w \begin{pmatrix} 1 \\ -\frac{8}{3} \\ 4/3 \\ 1 \end{pmatrix}$$

$\therefore \text{Ker}(A) = \text{gen}\left\{ \begin{pmatrix} 1 \\ -8/3 \\ 4/3 \\ 1 \end{pmatrix} \right\}$

$\text{null}(A) = 1$

c) Probaremos que $\text{Ran}(A) + \text{Null}(A) = n$

$\text{Ran}(A) + \text{Null}(A) = 4 \Rightarrow 3 + 1 = 4$ ✓