Encuentre la representación matricial A_T de la transformación lineal T, nu T, Im T, $\nu(T)$, y $\rho(T)$. A menos que se especifique otra cosa, suponga que B_1 y B_2 son bases canónicas.

15. $T: \mathbb{R}^2 \to \mathbb{R}^3$;

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x-y \\ 2x+y \\ y \end{bmatrix}; \quad B_1 = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}; \quad B_2 = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix} \right\}$$

Aplicamos T en los elementos de la base B_1.

$$T\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}; \qquad T\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$$

de esta manera tenemos que.

$$A_T = \begin{bmatrix} 1 & -1 \\ 5 & 4 \\ 1 & 2 \end{bmatrix}$$

Para determinar la base de la imagen multiplicamos

$$\begin{bmatrix} 1 & -1 \\ 5 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Reducimos con Gauss

$$\left[\begin{array}{cc|c} 1 & -1 & a \\ 5 & 4 & b \\ 1 & 2 & c \end{array}\right] \underset{\substack{F_2 - 5F_1 \\ F_3 - F_1}}{\sim} \left[\begin{array}{cc|c} 1 & -1 & a \\ 0 & 9 & b-5a \\ 0 & 3 & c-a \end{array}\right] \underset{R_2 \cdot \frac{1}{3}}{\sim} \left[\begin{array}{cc|c} 1 & -1 & a \\ 0 & 3 & \frac{b-5a}{3} \\ 0 & 3 & c-a \end{array}\right]$$

$$\overset{R_3 - R_2}{\sim} \left[\begin{array}{cc|c} 1 & -1 & a \\ 0 & 3 & \frac{b-5a}{3} \\ 0 & 0 & (c-a) - \left(\frac{b-5a}{3}\right) \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & a \\ 0 & 3 & \frac{b-5a}{3} \\ 0 & 0 & \frac{3c+2a-b}{3} \end{array}\right]$$

$\Rightarrow \dfrac{3c+2a-b}{3} = 0$ si $a = x$, $b = y$, $c = z$

$\Rightarrow \dfrac{3z+2x-y}{3} = 0 \Rightarrow 3z+2x-y = 3(0) = 0$

$\dim[\mathrm{Im}(T)] = 2.$

$\mathrm{Im}(T) = \{(x, y, z) \in \mathbb{R}^3 \mid 3z+2x-y = 0\}$

Ahora sacaremos el núcleo.

$$\left[\begin{array}{cc:c} 1 & -1 & 0 \\ 5 & 4 & 0 \\ 1 & 2 & 0 \end{array}\right] \begin{array}{l} \\ F_2-5F_1 \\ F_3-F_1 \end{array} \sim \left[\begin{array}{cc:c} 1 & -1 & 0 \\ 0 & 9 & 0 \\ 0 & 3 & 0 \end{array}\right] F_2 \cdot \tfrac{1}{3} \sim \left[\begin{array}{cc:c} 1 & -1 & 0 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \end{array}\right]$$

$$\underset{F_3-F_2}{\sim} \left[\begin{array}{cc:c} 1 & -1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} 3y = 0 \\ y = 0/3 \\ y = 0 \end{array} \qquad \begin{array}{l} x - y = 0 \\ x - 0 = 0 \\ x = 0 \end{array}$$

$nu(T) = \{(0,0)\}$

$\nu(T) =$ Nulidad $\longrightarrow \dim[nu(T)] = 0$

$\rho(T) =$ Rango $\longrightarrow \dim[Im(T)] = 2.$

Tenemos que

$$\dim \mathbb{R}^2 = \dim[nu(T)] + \dim[Im(T)]$$

$$\dim \mathbb{R}^2 - \dim[nu(T)] = \dim[Im(T)]$$

$$\therefore \quad 2 - 0 = 2$$

$$\dim[Im(T)] = 2.$$