(9.9.1, 41)

Calcule un vector fijo de probabilidad para la matriz

$$\begin{pmatrix} \frac{2}{3} & 1/6 & 1/6 \\ 1/4 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{pmatrix}$$

Se busca un vector de probabilidad t tal que $tT = t$. Si $t = (x, y, z)$ resolveremos la ecuación

$$(x, y, z)\begin{pmatrix} 2/3 & 1/6 & 1/6 \\ 1/4 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{pmatrix} = (x, y, z)$$

$$\Rightarrow \left[\frac{2}{3}x + \frac{1}{4}y,\ \frac{1}{6}x + \frac{1}{2}y,\ \frac{1}{6}x + \frac{1}{4}y + z\right] = (x, y, z)$$

$$\Rightarrow \begin{matrix} \frac{2}{3}x + \frac{1}{4}y = x \\ \frac{1}{6}x + \frac{1}{2}y = y \\ \frac{1}{6}x + \frac{1}{4}y + z = z \end{matrix} \quad \Rightarrow \quad \begin{matrix} -\frac{1}{3}x + \frac{1}{4}y = 0 \\ \frac{1}{6}x - \frac{1}{2}y = 0 \\ \frac{1}{6}x + \frac{1}{4}y = 0 \end{matrix}$$

Como t es un vector de probabilidad deberemos tener que $(x + y + z = 1$. Esto nos lleva al sistema

$$\begin{matrix} -\frac{1}{3}x + \frac{1}{4}y = 0 \\ \frac{1}{6}x - \frac{1}{2}y = 0 \\ \frac{1}{6}x + \frac{1}{4}y = 0 \\ x + y + z = 1 \end{matrix}$$

$\Rightarrow$

$$\left(\begin{array}{ccc|c} -1/3 & 1/4 & 0 & 0 \\ 1/6 & -1/2 & 0 & 0 \\ 1/6 & 1/4 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right) R_1 / -\tfrac{1}{3} \sim R_1 \left(\begin{array}{ccc|c} 1 & -\frac{3}{4} & 0 & 0 \\ 1/6 & -1/2 & 0 & 0 \\ 1/6 & 1/4 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right) R_2 - \tfrac{1}{6}R_1 \sim R_2$$

$$\left(\begin{array}{ccc|c} 1 & -3/4 & 0 & 0 \\ 0 & -3/8 & 0 & 0 \\ 1/6 & 1/4 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right) R_3 - \tfrac{1}{6}R_1 \sim R_3 \left(\begin{array}{ccc|c} 1 & -3/4 & 0 & 0 \\ 0 & -3/8 & 0 & 0 \\ 0 & 3/8 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right) R_4 - R_1 \sim R_4$$

$$\left(\begin{array}{ccc|c} 1 & -3/4 & 0 & 0 \\ 0 & -3/8 & 0 & 0 \\ 0 & 3/8 & 0 & 0 \\ 0 & 7/4 & 1 & 1 \end{array}\right) R_2 / -\tfrac{3}{8} \sim R_2 \left(\begin{array}{ccc|c} 1 & -3/4 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 3/8 & 0 & 0 \\ 0 & 7/4 & 1 & 1 \end{array}\right) R_3 - \tfrac{3}{8}R_2 \sim R_3$$

$$\left(\begin{array}{ccc|c} 1 & -3/4 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 7/4 & 1 & 1 \end{array}\right) R_4 - \tfrac{7}{4}R_2 \sim R_4 \left(\begin{array}{ccc|c} 1 & -3/4 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) R_1 - 3/4 R_2 \sim R_1$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) R_3 \leftrightarrow R_4 \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{matrix} x = 0 \\ y = 0 \\ z = 1 \end{matrix}$$

$\therefore$ El único vector de probabilidad (t) es $(0, 0, 1)$