

$$(4, 45, 18)$$

Find a basis for the subspace of \mathbb{R}^4 that is spanned by the vectors

$$V_1 = (1, 1, 1, 1) \quad V_2 = (2, 2, 2, 0) \quad V_3 = (0, 0, 0, 3) \quad V_4 = (3, 3, 3, 4)$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 3 & 3 & 3 & 4 \end{pmatrix} \xrightarrow{-2R_1 + R_2 \sim R_2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 3 \\ 3 & 3 & 3 & 4 \end{pmatrix} \xrightarrow{-3R_1 + R_4 \sim R_4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} R_2 / -2$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{-3R_2 + R_3 \sim R_3} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_4 - R_2 \sim R_4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} R_1 - R_2 - R_4$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Podemos ver que una base es } (1, 1, 1, 0), (0, 0, 0, 1) \text{ de dim} = 2$$