2.1

a) $0 \in H$

Tenemos $H = P_n(x)$, con $m < n$ y $P_m(x) = 0$.

$\Rightarrow$ tomamos $P_m(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 +, \dots + a_{n-1} x^{n-1}$

entonces $0 \in H$ dado que $a = 0$

b) $u, v \in H \Rightarrow u + v \in H$

tomamos $u = P_{n-1}(x)^{-1} =$

Sea: $u = P_{n-1}(x^{n-1}) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 +, \dots + a_{n-1} x^{n-1}$.

y $v = g_{m-1}(x^{m-1}) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 +, \dots + b_{n-1} x^{n-1}$

entonces tenemos

$u + v = a_0 + b_0 + (a_1 + b_1) x + (a_2 + b_2) x^2 + (a_3 + b_3) x^4 + (a_{n-1} + b_{n-1}) x^{n-1}$

entonces $u + v = P_{n-1}(x^{n-1}) + S_{m-1}(x^{m-1}) \in H$

c) $u \in H$ y $\alpha$, escalar $\Rightarrow \alpha u \in H$

Sea $u = P_{n-1}(x)$

$\alpha \cdot (a_0 + a_1 x + a_2 x^2 + a_3 x^4, + \dots + a_{n-1} x^{n-1} = (\alpha a_0 + (\alpha a_1) x +$

$(\alpha a_2) x^2 + (\alpha a_3) x^3 + (\alpha a_4) x^4 +, \dots + (\alpha a_{n-1}) x^{n-1}$

$\therefore \alpha u = \alpha P_{n-1}(x) \in H$, con $H < V$

es subespacio de V.

$\therefore$ H es subespacio vectorial